Theory Predictions for Neutrino Mixing Parameters

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Neutrino Mass beyond the SM

- SM: effective low energy theory
- new physics effects suppressed by powers of new physics scale M

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{\mathcal{O}_{5D}}{M} + \frac{\mathcal{O}_{6D}}{M^2} + \ldots$$

new physics effects

- neutrino masses generated by dim-5 operators

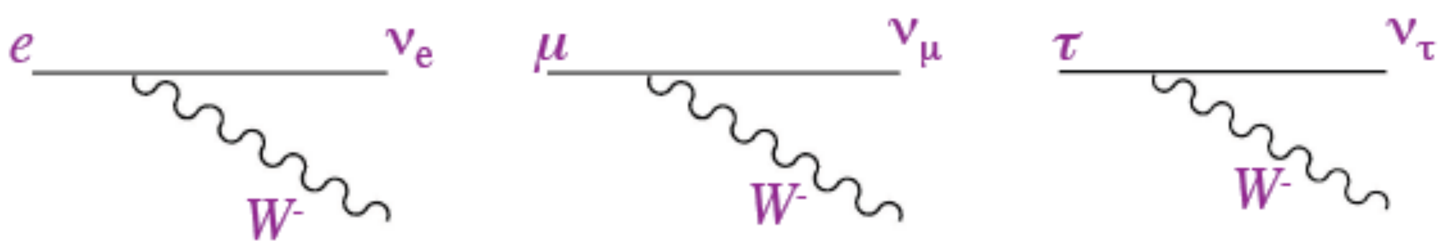
$$\frac{\lambda_{ij}}{M} H H L_i L_j \quad \Rightarrow \quad m_\nu = \lambda_{ij} \frac{v^2}{M}$$

λ_{ij} are dimensionless couplings;
M is some high scale

- high M ⇒ small m_ν
- total lepton number and individual family lepton numbers broken
  - lepton mixing expected
  - $\mu \rightarrow e \gamma$ (MEG @ PSI, ...) ; μ - e conversion (Mu2e @ Fermilab) ;
    $\tau \rightarrow \mu \gamma$, $\tau \rightarrow e \gamma$ decays (SuperB, LHCb)

What if Neutrinos Have Mass?

- Similar to the quark sector, there can be mismatch between mass eigenstates and weak eigenstates
- weak interactions eigenstates: ν_e, ν_μ, ν_τ



- mass eigenstates: ν_1, ν_2, ν_3
- Pontecorvo-Maki-Nakagawa-Sakata (PMNS) Matrix

$$\begin{aligned} V_{e,R}^{\dagger} M_e V_{e,L} &= \mathrm{diag}(m_e, m_\mu, m_\tau) \\ V_{\nu,L}^{T} M_\nu V_{\nu,L} &= \mathrm{diag}(m_1, m_2, m_3) \end{aligned}$$

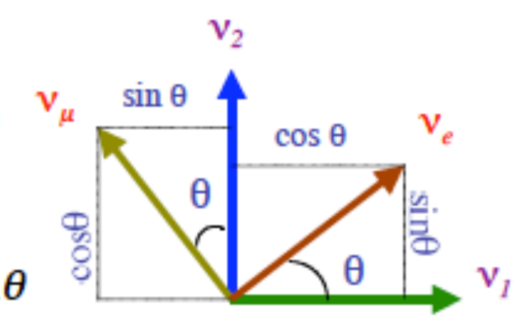
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\boxed{U_{MNS} = V_{e,L}^{\dagger} V_{\nu,L}}$$

Oscillation Mechanism

- Simplified two-flavor analysis:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$



$$|\nu_e\rangle = |\nu_1\rangle\cos\theta + |\nu_2\rangle\sin\theta$$

$$|\nu_\mu\rangle = |\nu_2\rangle\cos\theta - |\nu_1\rangle\sin\theta$$

- In vacuum: $|\nu_\mu\rangle$ evolves in time

$$|\nu_\mu(t)\rangle = |\nu_2\rangle e^{-im_2^2 t/4p}\cos\theta - |\nu_1\rangle e^{-im_1^2 t/4p}\sin\theta$$

- transition probability from ν_μ to ν_e

$$P(\nu_\mu \to \nu_e) = \langle \nu_e | \nu_\mu(t) \rangle$$
$$= \sin^2 2\theta \, \sin^2(\pi L/\lambda)$$

- Survival probability for ν_μ

$$P(\nu_\mu \to \nu_\mu)$$
$$= 1 - \sin^2 2\theta \, \sin^2(\pi L/\lambda)$$

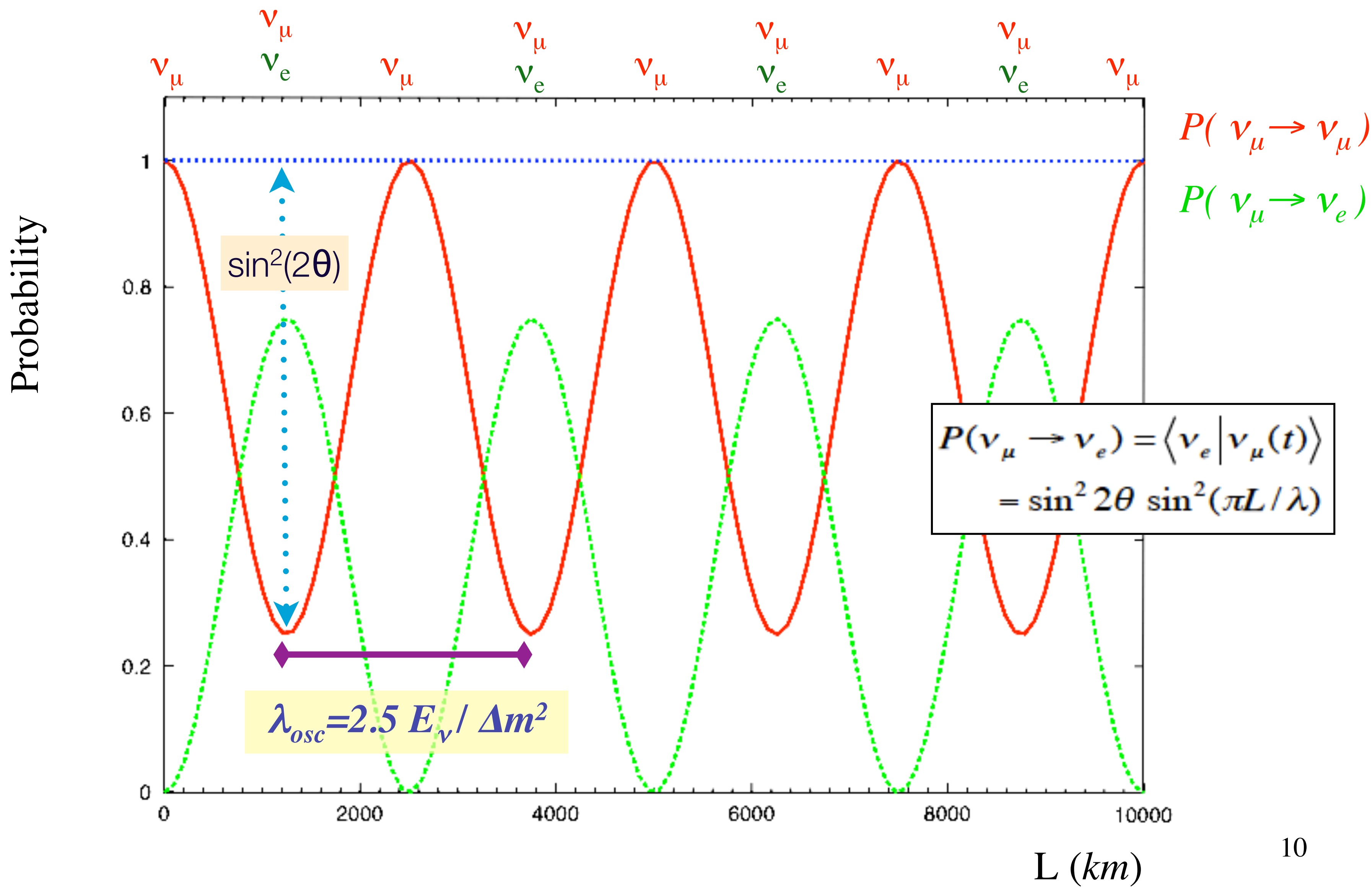
- Oscillation length

$$\lambda = \frac{2.5 E_\nu}{\Delta m^2}$$

$$\Delta m^2 = m_1^2 - m_2^2$$

Δm^2 must be non-zero to have neutrino oscillation!!

Vacuum oscillation: $E_\nu = 1$ GeV, $\Delta m^2 = 10^{-3}$ eV^2, $\theta = \pi/6$



$$P(\nu_\mu \to \nu_e) = \langle \nu_e | \nu_\mu(t) \rangle = \sin^2 2\theta \, \sin^2(\pi L/\lambda)$$

Leptonic Mixing Matrix

- Three neutrino case:
  - two mass differences: $\Delta m_a^2,\ \Delta m_s^2$

$$U_{MNS} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_a & s_a \\ 0 & -s_a & c_a \end{bmatrix} \begin{bmatrix} c_x & 0 & s_x e^{-i\delta} \\ o & 1 & o \\ -s_x e^{i\delta} & 0 & c_x \end{bmatrix} \begin{bmatrix} c_s & s_s & 0 \\ -s_s & c_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & o & 0 \\ 0 & e^{i\left(\frac{1}{2}\phi_{12}\right)} & 0 \\ 0 & 0 & e^{i\left(\frac{1}{2}\phi_{13}+\delta\right)} \end{bmatrix}$$

atm reactor solar Majorana phases

  - three mixing angles: $c_a,\ c_s,\ c_x$
  - three (one) CP phases for Majorana (Dirac) case: $\delta,\ \phi_{12},\ \phi_{13}$
    - CP violation in neutrino oscillation sensitive to Dirac phase: δ
    - neutrinoless double beta decay sensitive to Majorana phases: $\Phi_{12},\ \Phi_{13}$

Leptonic Mixing Matrix

measurements in precision phase

- Three neutrino case:
  - two mass differences: $\Delta m_a^2, \ \Delta m_s^2$

$$U_{MNS} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_a & s_a \\ 0 & -s_a & c_a \end{bmatrix}}_{\text{atm}} \underbrace{\begin{bmatrix} c_x & 0 & s_x e^{-i\delta} \\ 0 & 1 & 0 \\ -s_x e^{i\delta} & 0 & c_x \end{bmatrix}}_{\text{reactor}} \underbrace{\begin{bmatrix} c_s & s_s & 0 \\ -s_s & c_s & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{solar}} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\left(\frac{1}{2}\phi_{12}\right)} & 0 \\ 0 & 0 & e^{i\left(\frac{1}{2}\phi_{13}+\delta\right)} \end{bmatrix}}_{\text{Majorana phases}}$$

  - three mixing angles: $c_a, \ c_s, \ c_x$
  - three CP phases: $\delta, \ \phi_{12}, \ \phi_{13}$
    - CP violation in neutrino oscillation sensitive to Dirac phase: δ
    - neutrinoless double beta decay sensitive to Majorana phases: $\Phi_{12}, \ \Phi_{13}$

Leptonic Mixing Matrix

Discovery Phase
for theta13

- Three neutrino case:
  - two mass differences: $\Delta m_a^2, \; \Delta m_s^2$

$$U_{MNS} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_a & s_a \\ 0 & -s_a & c_a \end{bmatrix} \begin{bmatrix} c_x & 0 & s_x e^{-i\delta} \\ 0 & 1 & 0 \\ -s_x e^{i\delta} & 0 & c_x \end{bmatrix} \begin{bmatrix} c_s & s_s & 0 \\ -s_s & c_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\left(\frac{1}{2}\phi_{12}\right)} & 0 \\ 0 & 0 & e^{i\left(\frac{1}{2}\phi_{13}+\delta\right)} \end{bmatrix}$$

atm   reactor   solar   Majorana phases

  - three mixing angles: $c_a, \; c_s, \; c_x$
  - three CP phases: $\delta, \; \phi_{12}, \; \phi_{13}$
    - CP violation in neutrino oscillation sensitive to Dirac phase: δ
    - neutrinoless double beta decay sensitive to Majorana phases: $\Phi_{12}, \; \Phi_{13}$

Where Do We Stand?

- Exciting Time in ν Physics: recent hints of large θ_{13} from T2K, MINOS, Double Chooz, Daya Bay, RENO
- Latest 3 neutrino global analysis (including T2K and MINOS):

$$P(\nu_a \to \nu_b) = |\langle \nu_b | \nu,\ t\rangle|^2 \simeq \sin^2 2\theta\ \sin^2\left(\frac{\Delta m^2}{4E} L\right)$$

Fogli, Lisi, Marrone, Palazzo, Rotunno, arXiv:1106.6028
(see also, Schwetz, Tortola, Valle, arXiv:1108.1376)

Parameter	$\delta m^2/10^{-5}$ eV^2	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$	$\Delta m^2/10^{-3}$ eV^2
Best fit	7.58	0.306	0.021	0.42	2.35
		(0.312)	(0.025)		
1σ range	7.32 – 7.80	0.291 – 0.324	0.013 – 0.028	0.39 – 0.50	2.26 – 2.47
		(0.296 – 0.329)	(0.018 – 0.032)		
2σ range	7.16 – 7.99	0.275 – 0.342	0.008 – 0.036	0.36 – 0.60	2.17 – 2.57
		(0.280 – 0.347)	(0.012 – 0.041)		
3σ range	6.99 – 8.18	0.259 – 0.359	0.001 – 0.044	0.34 – 0.64	2.06 – 2.67
		(0.265 – 0.364)	(0.005 – 0.050)		

Current Global Fit: $\theta_{13} \neq 0$ at 3σ

Caution!! Different global fit analyses assume different error correlations among experiments ⇒ different results

Where Do We Stand?

- Global Fit Results at 1σ (3σ):

Fogli, Lisi, Marrone, Palazzo, Rotunno, arXiv:1106.6028
(see also, Schwetz, Tortola, Valle, arXiv:1108.1376)

$$\theta_{12} = 34.0^{\circ}{}^{+1.0^\circ}_{-0.9^\circ}\ {}^{(+2.9^\circ)}_{(-2.7^\circ)}\ , \qquad \theta_{23} = 46.1^{\circ}{}^{+3.5^\circ}_{-4.0^\circ}\ {}^{(+7.0^\circ)}_{(-7.5^\circ)}\ , \qquad \theta_{13} = \begin{cases} 6.5^{\circ}{}^{+1.6^\circ}_{-1.4^\circ}\ {}^{(+4.2^\circ)}_{(-4.7^\circ)}\ , & \mathrm{NH} \\ 7.3^{\circ}{}^{+1.7^\circ}_{-1.5^\circ}\ {}^{(+4.1^\circ)}_{(-5.5^\circ)}\ , & \mathrm{IH} \end{cases}$$

$$\Delta m^2_{21}[10^{-5}\mathrm{eV}^2] = 7.59^{+0.20}_{-0.18}\ {}^{(+0.60)}_{(-0.50)}\ , \qquad \Delta m^2_{31}[10^{-3}\mathrm{eV}^2] = \begin{cases} 2.50^{+0.09}_{-0.16}\ {}^{(+0.26)}_{(-0.36)}\ , & \mathrm{NH} \\ 2.40^{+0.08}_{-0.09}\ {}^{(+0.27)}_{(-0.27)}\ , & \mathrm{IH} \end{cases}$$

- Combining Results from T2K, MINOS, Double CHOOZ, Daya Bay, RENO Experimets:

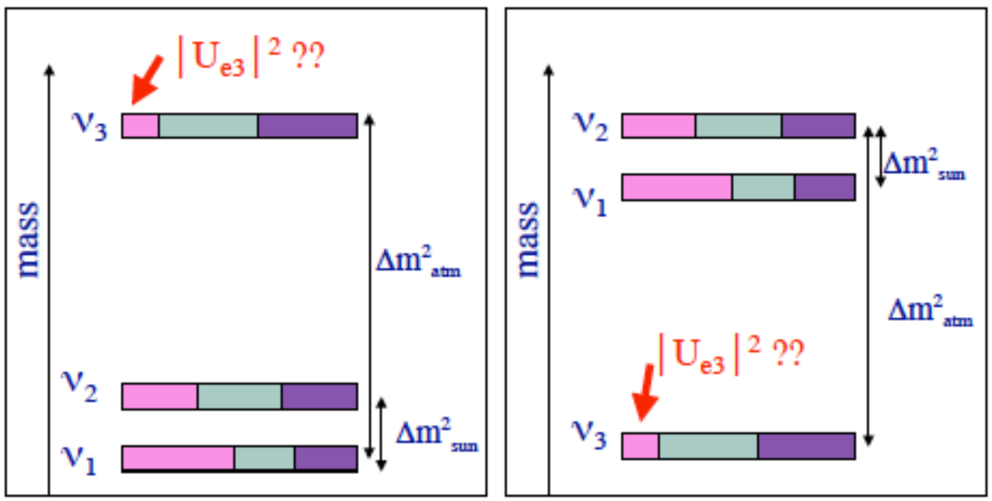
Machado, Minakata, Nunokawa, Zukanovich Funchal,
arXiv:1111.3330v4 (last update: 04/28/2012)

$$\sin^2 2\theta_{13} = 0.096 \pm 0.013(\pm 0.040)\ \text{at}\ 1\ \sigma\ (3\ \sigma)\ \mathrm{CL}$$

Where Do We Stand?

- The known knowns:

normal hierarchy: inverted hierarchy:



What's Next?
Reactor Exp: Double Chooz, Daya Bay, Reno
Long Baseline Exp: MINOS, NOvA, T2K, LBNE...

The known unknowns:

- Precisely how "large" is θ_{13}?
  (ν_e component of ν_3)
- $\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, $\theta_{23} = \pi/4$?
  (ν_3 composition of ν)
- neutrino mass hierarchy (Δm_{13}^2)?
- CP violation in neutrino oscillations?
- Majorana vs Dirac?

The unknown unknowns?

Theoretical Challenges

(i) Absolute mass scale: Why $m_\nu << m_{u,d,e}$?

- seesaw mechanism: most appealing scenario ⇒ Majorana
  - GUT scale (type-I, II) vs TeV scale (type-III, double seesaw)
- TeV scale new physics (extra dimension, U(1)´, ...) ⇒ Dirac or Majorana

(ii) Flavor Structure: Why neutrino mixing large while quark mixing small?

- seesaw doesn't explain entire mass matrix w/ 2 large, 1 small mixing angles
- neutrino anarchy: no parametrically small number — Hall, Murayama, Weiner (2000); de Gouvea, Murayama (2003)
  - near degenerate spectrum, large mixing
  - lack of predictivity
  - still alive and kicking! — de Gouvea, Murayama, arXiv;1204.1249
- family symmetry: there's a structure, expansion parameter (~~symmetry~~ effect)
  - mixing result from dynamics of underlying symmetry
  - quark-lepton connection ↔ GUT : can be highly predictive

- In this talk: assume 3 generations, no LSND/MiniBoone/Reactor Anomaly
  - sterile neutrinos: tension between fit to oscillation data and cosmology

Small Neutrino Mass: Seesaw Mechanism

Yanagida, 1979; Gell-Mann, Ramond, Slansky, 1979; Mohapatra, Senjanovic, 1981

- Mixture of light fields and heavy fields

$$\begin{pmatrix} \nu_L & \nu_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

ν_R: sterile (singlet under ALL gauge groups in SM)

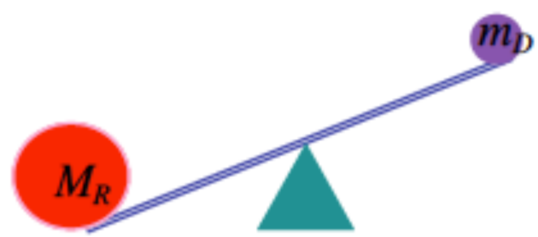
$\nu_R \nu_R$ mass term allowed

- Diagonalize the mass matrix:

$$m_\nu \sim m_{light} \sim \frac{m_D^2}{M_R} << m_D$$

$$m_{heavy} \sim M_R$$

- Smallness of neutrino masses suggest a high mass scale



For $m_{\nu_3} \sim \sqrt{\Delta m^2_{atm}}$

If $m_D \sim m_t \sim 180 \; GeV$

⇒ $M_R \sim 10^{15} \; GeV$ (GUT !!)

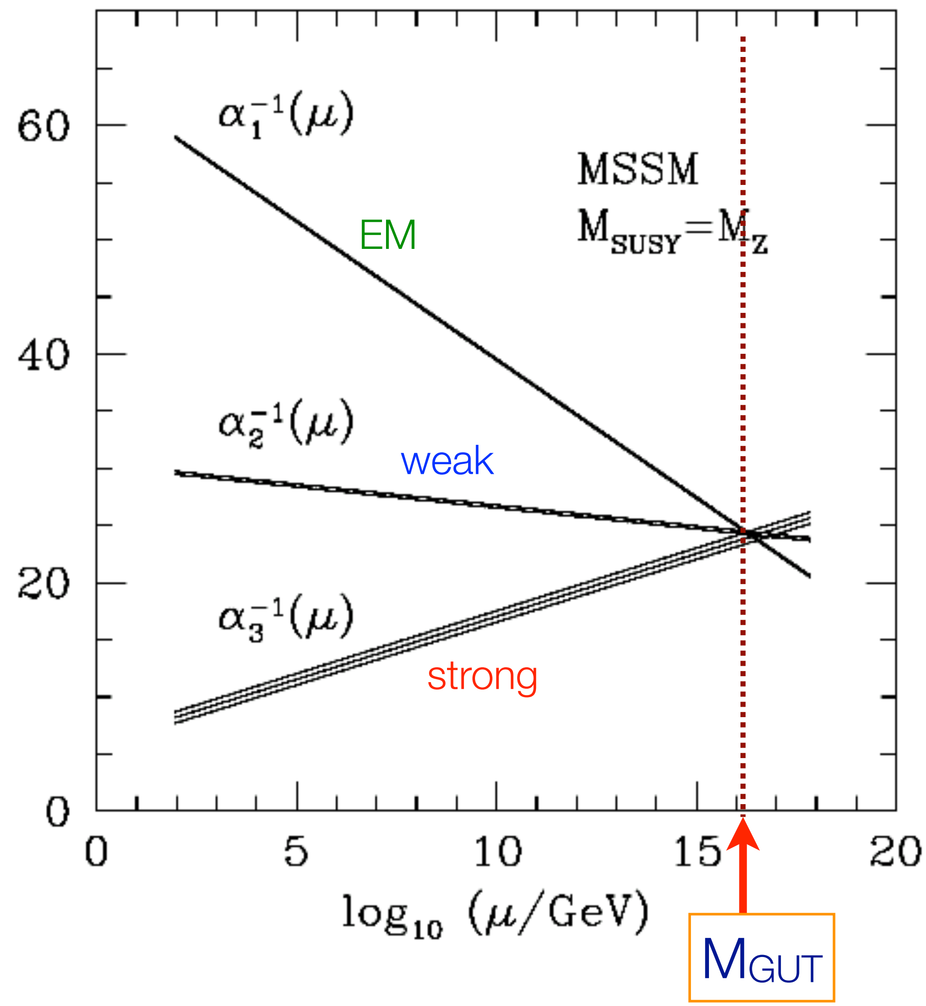
Grand Unification

- Motivations:
  - Electromagnetic, weak, and strong forces have very different strengths
  - But their strengths become the same at 10^{16} GeV if there is supersymmetry
  - To obtain

$$m_\nu \sim (\Delta m^2_{atm})^{1/2}, \; m_D \sim m_{top}$$

$$M_R \sim 10^{15} \text{ GeV}$$

coupling constants run!



Origin of Flavor Mixing and Mass Hierarchy

- SM: 22 arbitrary parameters in Yukawa sector
- No fundamental origin found or suggested
- Reduce number of parameters
    - Grand Unification
        - seesaw scale ~ GUT scale
        - quarks and leptons unified
        - 1 coupling for entire multiplet

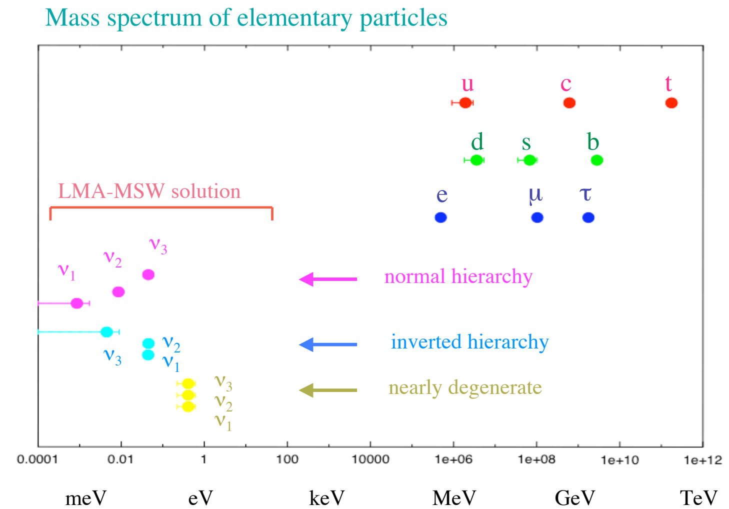
    ⇒ intra-family relations (e.g. SO(10))

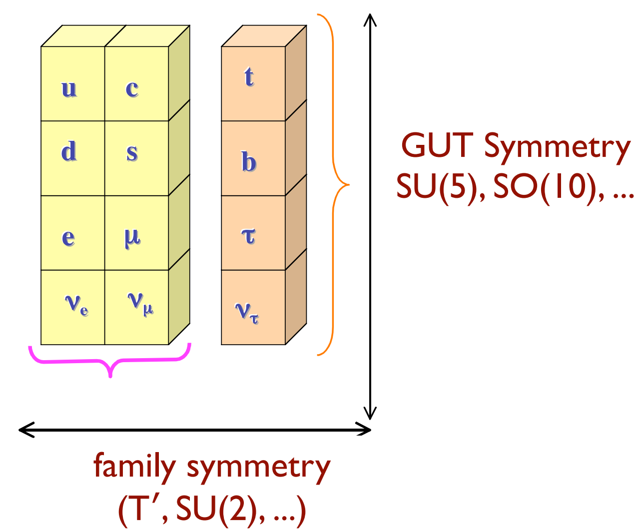
    Up-type quarks ⇔ Dirac neutrinos

    Down-type quarks ⇔ charged leptons

    - Family Symmetry

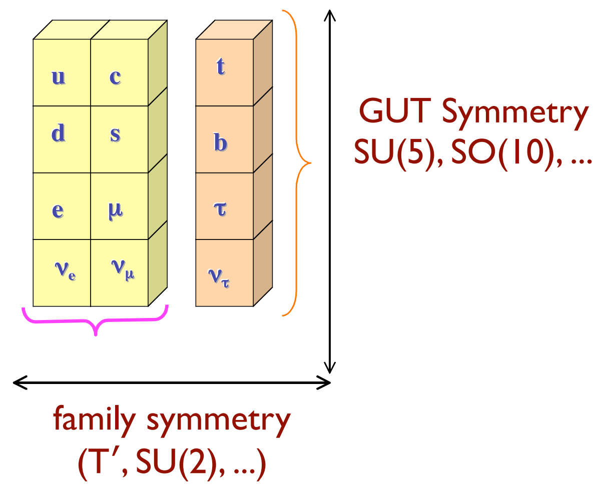
    ⇒ inter-family relations (flavor structure)





Origin of Flavor Mixing and Mass Hierarchy

- Several models have been constructed based on
  - GUT Symmetry [SU(5), SO(10)] ⊕ Family Symmetry G_F
- Family Symmetries G_F based on continuous groups:
  - U(1)
  - SU(2)
  - SU(3)



- Recently, models based on discrete family symmetry groups have been constructed
  - A_4 (tetrahedron)
  - T´ (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_4

Motivation: Tri-bimaximal (TBM) neutrino mixing

Discussion on Discrete gauge anomaly:
Araki, Kobayashi, Kubo, Ramos-Sanchez, Ratz, Vaudrevange (2008)

Tri-bimaximal Neutrino Mixing

- Neutrino Oscillation Parameters $P(\nu_a \to \nu_b) = \left|\langle \nu_b | \nu,\ t\rangle\right|^2 \simeq \sin^2 2\theta\ \sin^2\left(\frac{\Delta m^2}{4E} L\right)$

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Latest Global Fit (3σ)

Fogli, Lisi, Marrone, Palazzo, Rotunno, arXiv:1106.6028

$$\sin^2\theta_{atm} = 0.42\ (0.34 - 0.64)\ ,\ \sin^2\theta_{\odot} = 0.306\ (0.259 - 0.359)$$

$$\sin^2\theta_{13} = 0.021\ (0.001 - 0.044)$$

- Tri-bimaximal Mixing Pattern

Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\text{atm, TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\text{TBM}} = 1/3$$

$$\sin\theta_{13,\text{TBM}} = 0.$$

- Leading Order: TBM (from symmetry) + Corrections (dictated by symmetry)

Models for Tri-bimaximal Mixing

- Neutrino mass matrix

$$M = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} \longrightarrow \begin{aligned} \sin^2 2\theta_{23} &= 1 \\ \theta_{13} &= 0 \end{aligned}$$

solar mixing angle NOT fixed

μ-τ symmetry: Fukuyama, Nishiura; Mohapatra, Nussinov; Ma, Raidal; ...
S_3: Kubo, Mondragon, Mondragon, Rodriguez-Jauregui; Araki, Kubo, Paschos; Mohapatra, Nasri, Yu; ...
D_4: Ko, Kobayashi Park, Raby; Grimus, Lavoura; ...

- If A + B = C + D ⇒ $\tan^2\theta_{12} = 1/2$ TBM pattern
  - mass matrix M diagonalized by U_{TBM}

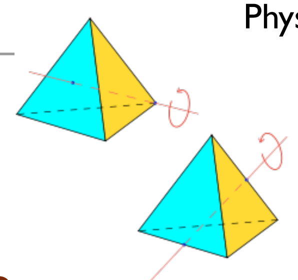
$$U_{TBM}^T \, M \, U_{TBM} = diag(m_1, m_2, m_3)$$

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

A_4: Ma, Rajasekaran; Babu, Ma, Valle; Altarelli, Feruglio; He, Keum, Volkas; ...
Z_3 x Z_7: Luhn, Nasri, Ramond; ...
S_4: Lam; Ketan Patel; Watanabe et al; ...
A_4 x S_3^4: Babu, Gabriel

An Example: a SUSY SU(5) x T´ Model

M.-C.C, K.T. Mahanthappa
Phys. Lett. B652, 34 (2007);
Phys. Lett. B681, 444 (2009)



- Double Tetrahedral Group T´
- Symmetries ⇒ 9 parameters in Yukawa sector ⇒ 22 physical observables
  - neutrino mixing angles from group theory (CG coefficients)
  - TBM: misalignment of symmetry breaking patterns
    - neutrino sector: T' → G_{TST2} , charged lepton sector: T' → G_T
  - GUT symmetry ⇒ deviation from TBM related to quark mixing θ_c
  - **complex CG's of T´ ⇒ Novel Origin of CP Violation** M.-C.C, K.T. Mahanthappa, Phys. Lett. B681, 444 (2009)
    - CP violation in both quark and lepton sectors entirely from group theory
    - connection between leptogenesis and CPV in neutrino oscillation
- family symmetry: forbid Higgsino mediated proton decay (a la Babu-Barr)

Predictions: a SUSY SU(5) x T´ Model

M.-C.C, K.T. Mahanthappa
Phys. Lett. B652, 34 (2007);
Phys. Lett. B681, 444 (2009)

- Charged Fermion Sector (7 parameters)

$$M_u = \begin{pmatrix} ig & \frac{1-i}{2}g & 0 \\ \frac{1-i}{2}g & g+(1-\frac{i}{2})h & k \\ 0 & k & 1 \end{pmatrix} y_t v_u$$

(k → V_{cb})

$$M_d,\ M_e^T = \begin{pmatrix} 0 & (1+i)b & 0 \\ -(1-i)b & (1,-3)c & 0 \\ b & b & 1 \end{pmatrix} y_b v_d \phi_0$$

(b → V_{ub})

spinorial representations in charged fermion sector ⇒ complex CGs

⇒ CPV in quark and lepton sectors

quark CP phase: γ = 45.6 degrees

Georgi-Jarlskog relations at GUT scale
⇒ $V_{d,L} \neq I$

$$m_d \simeq 3m_e \qquad m_\mu \simeq 3m_s$$

$$\theta_c \simeq \left| \sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c} \right| \sim \sqrt{m_d/m_s},$$

SU(5) ⇒ $M_d = (M_e)^T$

⇒ corrections to TBM related to θ_c

$$\theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3}\theta_c$$

Predictions: a SUSY SU(5) x T´ Model

M.-C.C, K.T. Mahanthappa
Phys. Lett. B652, 34 (2007);
Phys. Lett. B681, 444 (2009)

- Neutrino Sector (2 parameters):

$$M_{RR} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} S_0 \qquad M_D = \begin{pmatrix} 2\xi_0+\eta_0 & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & -\xi_0+\eta_0 \\ -\xi_0 & -\xi_0+\eta_0 & 2\xi_0 \end{pmatrix} \zeta_0\zeta_0' v_u$$

- Seesaw mechanism:

$$U_{TBM}^T M_\nu U_{TBM} = \mathrm{diag}((3\xi_0+\eta_0)^2, \eta_0^2, -(-3\xi_0+\eta_0)^2)\frac{(\zeta_0\zeta_0' v_u)^2}{s_0\Lambda}$$

- Prediction for MNS matrix:

$$U_{\rm MNS} = V_{e,L}^\dagger U_{\rm TBM} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\theta_{13} \simeq \theta_c/3\sqrt{2}$$

CGs of SU(5) & T´

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c\cos\delta$$

neutrino mixing angle

1/2

quark mixing angle

complex CGs: leptonic Dirac CPV

δ = 227 degrees

⇒ connection between leptogenesis & leptonic CPV at low energy

- sum rule among absolute masses:

$$m_2^2 - m_1^2 = (\eta_0^4 - (3\xi_0+\eta_0)^4)\frac{(\zeta_0\zeta_0' v_u)^2}{S_0} > 0$$

$$m_3^2 - m_1^2 = -24\eta_0\xi_0(9\xi_0^2+\eta_0^2)\frac{(\zeta_0\zeta_0' v_u)^2}{S_0}$$

normal hierarchy predicted

Numerical Results: Neutrino Sector

- Diagonalization matrix for charged leptons $\begin{pmatrix} 0.997e^{i177^o} & 0.0823e^{i131^o} & 1.31\times10^{-5}e^{-i45^o} \\ 0.0823e^{i41.8^o} & 0.997e^{i176^o} & 0.000149e^{-i3.58^o} \\ 1.14\times10^{-6} & 0.000149 & 1 \end{pmatrix}$

- MNS Matrix   Note that these predictions do NOT depend on η_0 and ξ_0

$$|U_{MNS}| = \begin{pmatrix} 0.838 & 0.542 & 0.0583 \\ 0.362 & 0.610 & 0.705 \\ 0.408 & 0.577 & 0.707 \end{pmatrix}$$

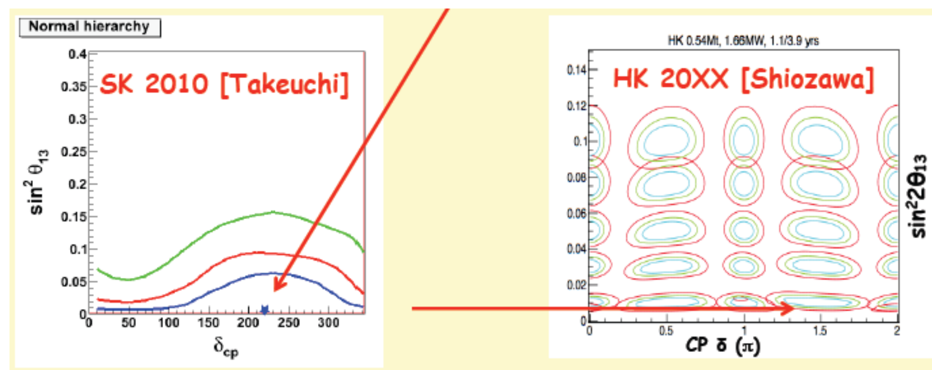
prediction for Dirac CP phase: δ = 227 degrees

$$\sin^2 2\theta_{atm} = 1, \quad \tan^2\theta_\odot = 0.419, \quad |U_{e3}| = 0.0583$$

$$J_\ell = -0.00967$$

low energy phases the only non-vanishing leptonic CPV phase

⇒ connection between leptogenesis & low energy CPV



SuperK best fit: δ = 220 degrees

- Neutrino Masses: using best fit values for Δm^2

$$\xi_0 = -0.0791\,, \quad \eta_0 = 0.1707\,, \qquad S_0 = 10^{12}\ \text{GeV}$$

2 independent parameters in neutrino sector

$$|m_1| = 0.00134\ \text{eV}, \ |m_2| = 0.00882\ \text{eV}, \ |m_3| = 0.0504\ \text{eV}$$

predicting: 3 masses, 3 angles, 3 CP Phases; both θ_{sol} & θ_{atm} agree with exp

- Majorana CPV invariants: $S_1 = -0.035\,, \ S_2 = -0.021$

Other Possibilities: Beyond TBM

- Current experimental precisions: TBM can be accidental

Albright, Rodejohann (2009); Abbas, Smirnov (2010)

- Other possibilities: e.g. Dodeca Mixing Matrix from D_{12} Symmetry

J. E. Kim, M.-S. Seo, (2010)

leading order:

$\theta_c = 15^o$, $\theta_{sol} = 30^o$, $\theta_{atm} = 45^o$

$$V_{\rm PMNS} = U_l^\dagger U_\nu = \begin{pmatrix} \cos\frac{\pi}{6} & \sin\frac{\pi}{6} & 0 \\ -\frac{1}{\sqrt{2}}\sin\frac{\pi}{6} & \frac{1}{\sqrt{2}}\cos\frac{\pi}{6} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\sin\frac{\pi}{6} & \frac{1}{\sqrt{2}}\cos\frac{\pi}{6} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\theta_c + \theta_{sol} = 45^o$ (not from GUT symmetry)

breaking of D_{12} :

$\theta_c = 15^o \rightarrow 13.4^o$

$\theta_{sol} = 30^o + O(\varepsilon)$, $\theta_{13} = O(\varepsilon)$

deviations correlated

Sum Rules: Quark-Lepton Complementarity

Quark Mixing

mixing parameters	best fit	3σ range
θ^q_{23}	2.36°	2.25° - 2.48°
θ^q_{12}	12.88°	12.75° - 13.01°
θ^q_{13}	0.21°	0.17° - 0.25°

Lepton Mixing

mixing parameters	best fit	3σ range
θ^e_{23}	42.8°	35.5° - 53.5°
θ^e_{12}	34.4°	31.5° - 37.6°
θ^e_{13}	5.6°	≤ 12.5°

- QLC-I (BM)
  $\theta_C + \theta_{sol} \cong 45^o$ Raidal, '04; Smirnov, Minakata, '04
  $\theta^q_{23} + \theta^e_{23} \cong 45^o$

measuring leptonic mixing parameters to the precision of those in quark sector

- QLC-II (TBM)
  $\tan^2\theta_{sol} \cong \tan^2\theta_{sol,TBM} + (\theta_C / 2) * \cos\delta_e$
  $\theta^e_{13} \cong \theta_C / 3\sqrt{2}$ Ferrandis, Pakvasa; King; Dutta, Mimura; M.-C.C., Mahanthappa

- testing sum rules: a *more* robust way to distinguish different classes of models

Conclusions

- Kudos to experimentalists on recent exciting theta13 results!
- efforts at current and future experiments important
  - fundamental properties of neutrinos
  - underlying new physics for neutrino mass and mixing
- Example: a SUSY SU(5) x T´ Model
  - GUT + Family Symmetries: 9 parameters ⇒ 22 physical observables
  - group theoretical origin of mixing
  - CP violation from complex CG coefficients
  - QLC sum rules:

quark CP phase: γ = 45.6 degrees

δ = 227 degrees

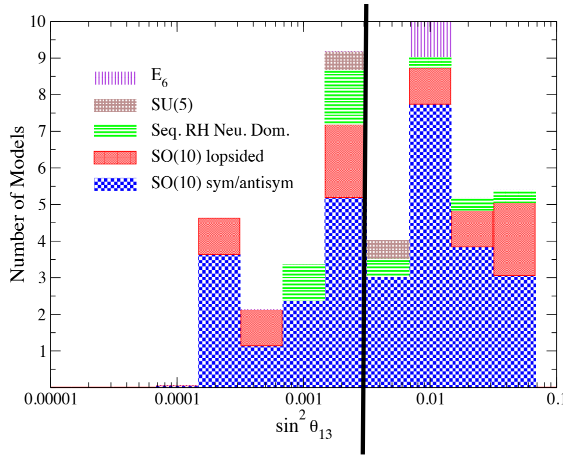
$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c \cos\delta$$

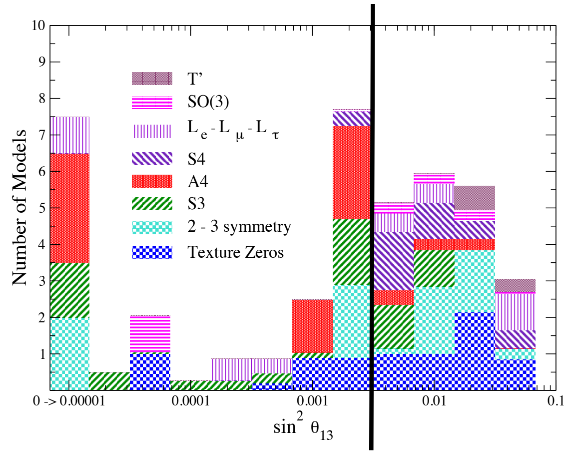
$$\theta_{13} \simeq \theta_c/3\sqrt{2}$$

  - normal hierarchy predicted

Conclusions

- precise measurements of oscillation parameters important for pinning down underlying new physics



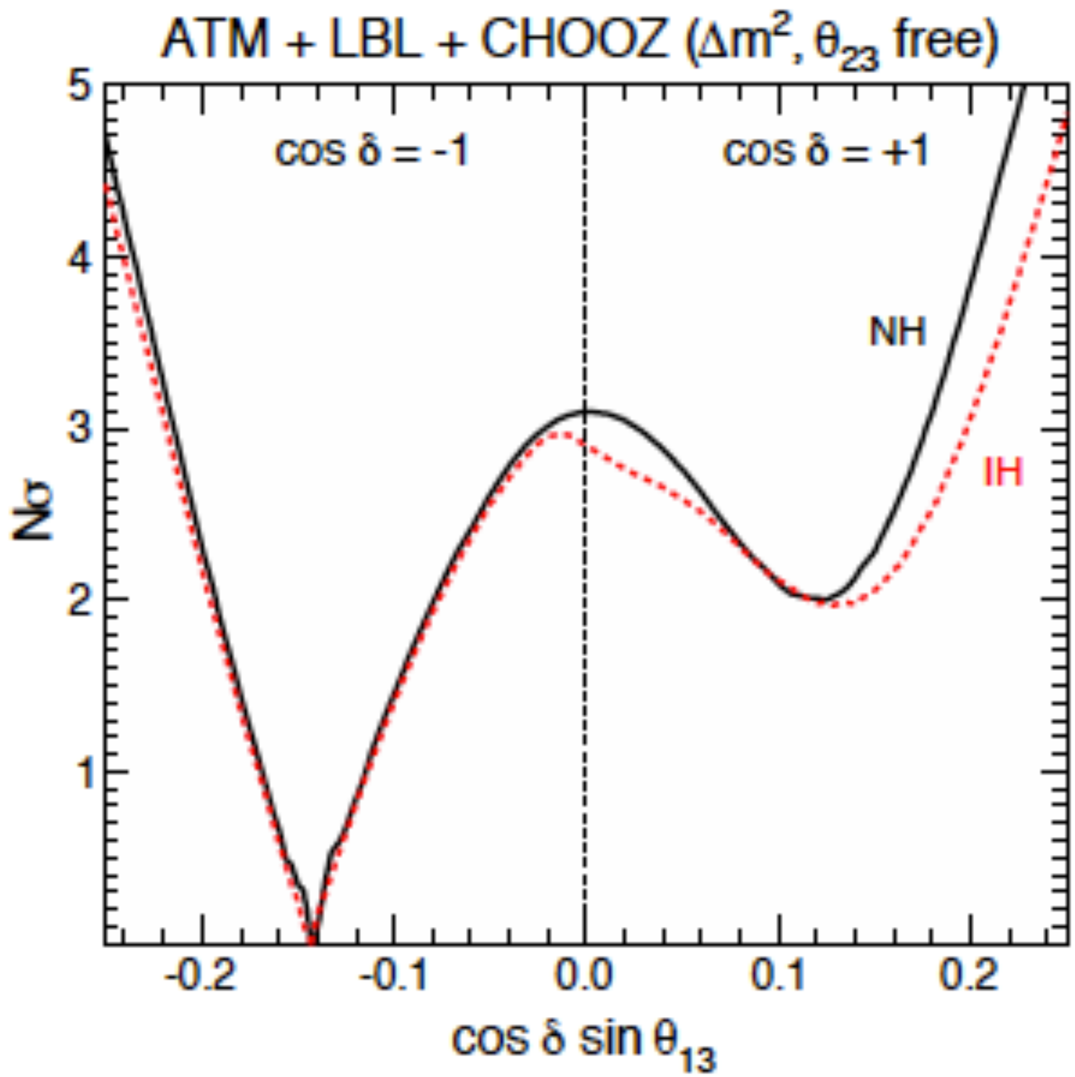


C.H. Albright (2009); updated from C. H. Albright, M.-C. C (2006)

- Theta13: Not merely measuring a number, but we are testing a paradigm
- Testing correlations: robust way to distinguish different classes of models
  - correlations among neutrino mixing parameters
  - sum rules among quark and lepton mixing parameters
  - correlations among other flavor violating processes

Backup Slides

Global Fit Including T2K/MINOS Results



Fogli, Lisi, Marrone, Palazzo, Rotunno, arXiv:1106.6028

Consistent with SuperK Best Fit: δ = 220 degrees (Neutrino 2010)

constraint on leptonic Dirac CP phase from global fit, albeit not statistically significant

Where Do We Stand?

- Search for absolute mass scale:
  - end point kinematic of tritium beta decays

$$m_{\nu_e} < 2.2 \text{ eV (95\% CL)} \quad \text{Mainz}$$
$$m_{\nu_\mu} < 170 \text{ keV}$$
$$m_{\nu_\tau} < 15.5 \text{ MeV}$$

KATRIN: increase sensitivity ~ 0.2 eV

$$\text{Tritium} \rightarrow He^3 + e^- + \overline{\nu}_e$$

  - WMAP + 2dFRGS + Lyα $\sum(m_{\nu_i}) < (0.36\text{-}1.5)$ eV Gonzalez-Garcia et al, arXiv:1006.3795
    - very model dependent
  - neutrinoless double beta decay
    - uncertainty in nuclear matrix element

GERDA: < (0.09-0.29) eV

$$\text{current bound:} \quad |\langle m \rangle| \equiv \left| \sum_{i=1,2,3} m_i U_{ie}^2 \right| < (0.19 - 0.68) \text{ eV (CUORICINO, Feb 2008)}$$

- Effective number of neutrinos:
  - WMAP7 + BAO: $N_{eff} = 4.34^{+0.86}_{-0.88}$ Komatsu et al, arXiv:1001.4538
  - BBN: $N_s < 1.2$ Mangano, Serpico, arXiv:1103.1261

resolved by Planck soon!