

Decaying vs Annihilating Dark Matter in Light of a Tentative Gamma-Ray Line

Mathias Garny (DESY Hamburg, Germany)



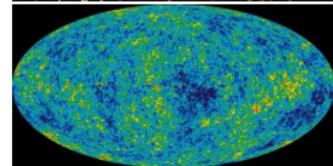
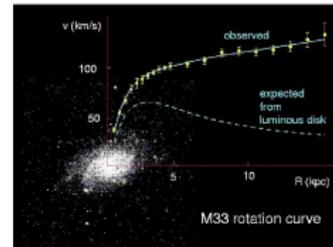
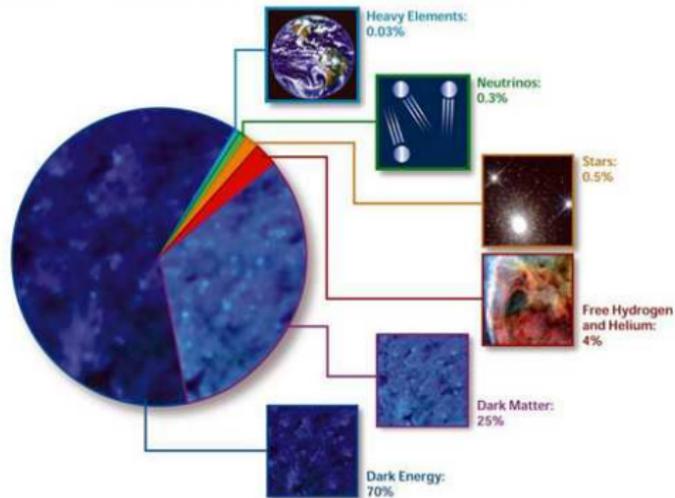
TUM, 05.07.12

based on arXiv:1206.7056 with Wilfried Buchmüller

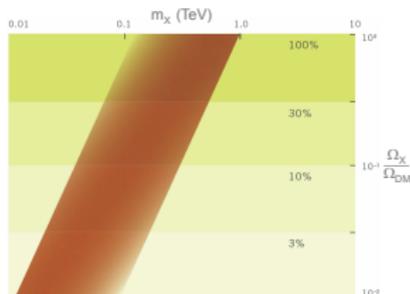
- Tentative Gamma Ray Line from Fermi LAT at $E_\gamma \sim 130$ GeV
- Models
 - Neutralino
 - Gravitino
- Constraints
 - Continuum Gamma Rays
 - Antiprotons
 - Morphology of the excess

The question about the actual existence and origin is under an active debate; in this talk, consequences of the hypothesis that dark matter is responsible will be discussed

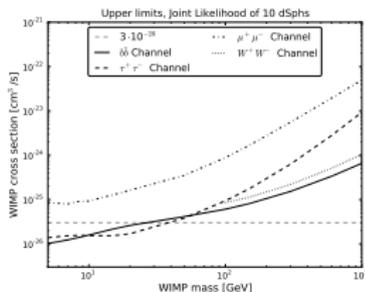
Standard Model of Cosmology



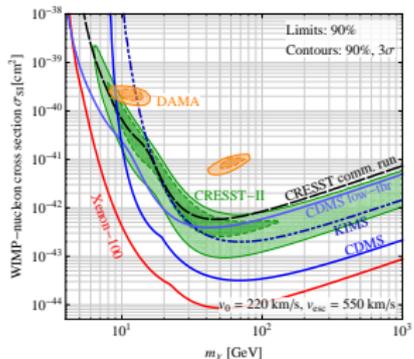
WIMP Dark Matter



Feng 2010

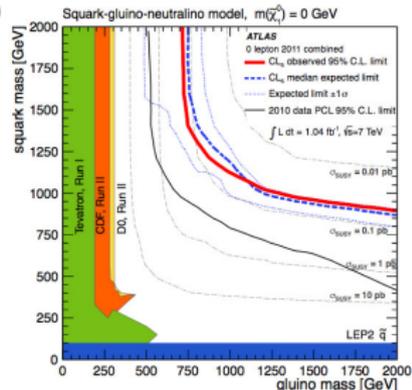
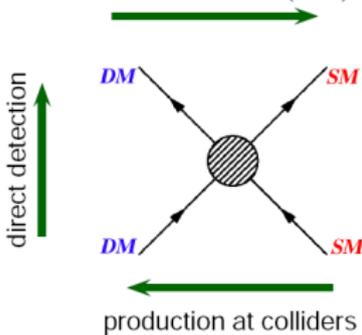


Fermi 1108.3546



Kopp, Schvez, Zupan 2011

thermal freeze-out (early Univ.)
indirect detection (now)



A Tentative Gamma-Ray Line from Dark Matter Annihilation at the Fermi Large Area Telescope

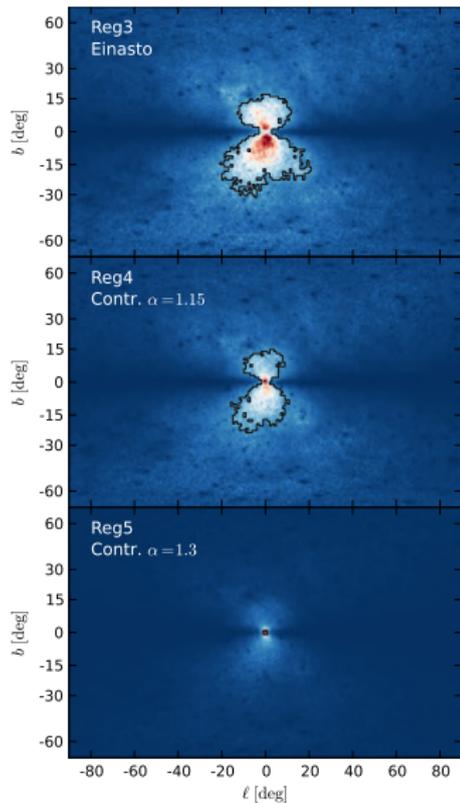
Christoph Weniger

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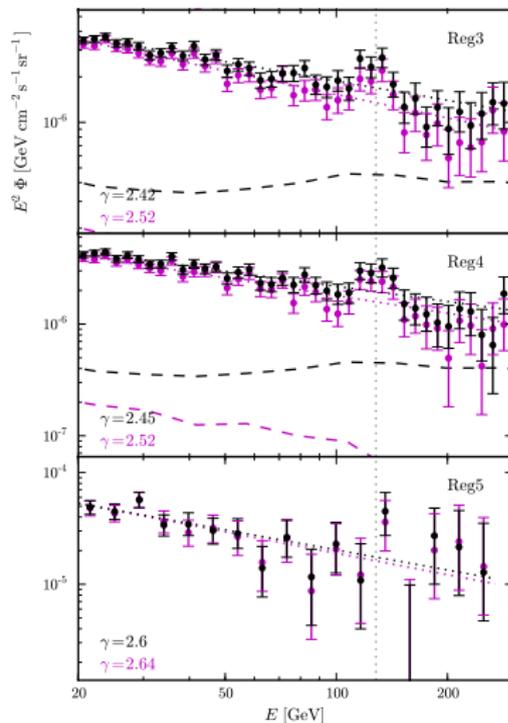
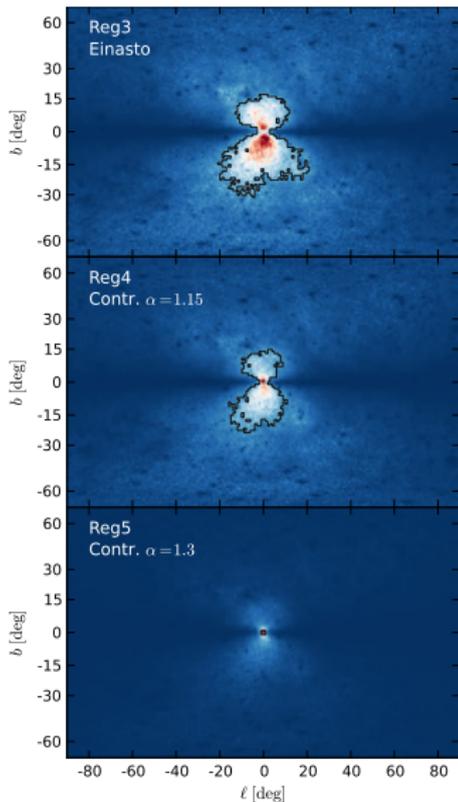
Abstract. The observation of a gamma-ray line in the cosmic-ray fluxes would be a smoking-gun signature for dark matter annihilation or decay in the Universe. We present an improved search for such signatures in the data of the Fermi Large Area Telescope (LAT), concentrating on energies between 20 and 300 GeV. Besides updating to 43 months of data, we use a new data-driven technique to select optimized target regions depending on the profile of the Galactic dark matter halo. In regions close to the Galactic center, we find a 4.6σ indication for a gamma-ray line at $E_\gamma \approx 130$ GeV. When taking into account the look-elsewhere effect the significance of the observed excess is 3.3σ . If interpreted in terms of dark matter particles annihilating into a photon pair, the observations imply a dark matter mass of $m_\chi = 129.8 \pm 2.4^{+7}_{-13}$ GeV and a partial annihilation cross-section of $\langle\sigma v\rangle_{\chi\chi\rightarrow\gamma\gamma} = (1.27 \pm 0.32^{+0.18}_{-0.28}) \times 10^{-27}$ cm³ s⁻¹ when using the Einasto dark matter profile. The evidence for the signal is based on about 50 photons; it will take a few years of additional data to clarify its existence.

Tentative Gamma Ray Line from Fermi LAT



Weniger 1204.2797

Tentative Gamma Ray Line from Fermi LAT



Weniger 1204.2797

Tentative Gamma Ray Line from Fermi LAT

DRAFT VERSION JUNE 15, 2012

Preprint typeset using L^AT_EX style emulatej v. 03/07/07

STRONG EVIDENCE FOR GAMMA-RAY LINE EMISSION FROM THE INNER GALAXY

MENG SU^{1,3}, DOUGLAS P. FINKBEINER^{1,2}

Draft version June 15, 2012

ABSTRACT

Using 3.7 years of *Fermi*-LAT data, we examine the diffuse 80 – 200 GeV emission in the inner Galaxy and find a resolved gamma-ray feature at $\sim 110 - 140$ GeV. We model the spatial distribution of this emission with a $\sim 3^\circ$ FWHM Gaussian, finding a best fit position 1.5° West of the Galactic Center. Even better fits are obtained for off-center Einasto and power-law profiles, which are preferred over the null (no line) hypothesis by 6.5σ ($5.0\sigma/5.4\sigma$ after trials factor correction for one/two line case) assuming an NFW density profile centered at $(\ell, b) = (-1.5^\circ, 0^\circ)$ with a power index $\alpha = 1.2$. The energy spectrum of this structure is consistent with a single spectral line (at energy 127.0 ± 2.0 GeV with $\chi^2 = 4.48$ for 4 d.o.f.). A pair of lines at 110.8 ± 4.4 GeV and 128.8 ± 2.7 GeV provides a marginally better fit (with $\chi^2 = 1.25$ for 2 d.o.f.). The total luminosity of the structure is $(3.2 \pm 0.6) \times 10^{35}$ erg/s, or $(1.7 \pm 0.4) \times 10^{36}$ photons/sec. The energies in the two-line case are compatible with a 127.3 ± 2.7 GeV WIMP annihilating through $\gamma\gamma$ and γZ (with $\chi^2 = 1.67$ for 3 d.o.f.). We describe a possible change to the *Fermi* scan strategy that would accumulate S/N on spectral lines in the Galactic center 4 times as fast as the current survey strategy.

Subject headings: gamma rays — diffuse emission — milky way — dark matter

1. INTRODUCTION

Although various cosmological and astrophysical observations provide compelling evidence for dark matter

ing one photon plus a Higgs boson, Z boson, or other chargeless non-SM particle. In most models, dark matter does not annihilate directly to photons, but in models

[astro-ph.HE] 14 Jun 2012

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Observable monochromatic photons from cosmic photino annihilation

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(Received 3 November 1987)

A recent suggestion that the observations of monochromatic photons from annihilation of photinos into quarkonium plus a photon could be a signature of dark-matter photinos is extended to include all *S*- and *P*-wave bound quarkonium states for a general interaction Lagrangian and an estimate of the cross sections is given. We also propose the process $\lambda\bar{\lambda} \rightarrow \gamma\gamma$ as a potentially rich source of monochromatic photons and estimate its strength.

I. INTRODUCTION

The dark-matter problem in the Universe has recently attracted attention among astrophysicists and particle physicists.¹ Much work has been devoted to the task of finding experimental signatures for the various candidates for dark matter—axions, photinos, Higgsinos, heavy neutrinos, etc.² In the case of photinos (λ) it has recently been suggested by Srednicki, Theisen, and Silk³ that the annihilation process $\lambda\bar{\lambda} \rightarrow V\gamma$ might give a detectable photon signal (V is the vector-meson bound state of heavy quarks, e.g., $c\bar{c}$). This analysis, which was based on a pointlike coupling between the $Q\bar{Q}$ pair and the vector meson, has been criticized by Rudaz⁴ who found a lower branching ratio for $\lambda\bar{\lambda} \rightarrow V\gamma$ using a more realistic bound-state description of the vector meson. For $m_\lambda < 4$ GeV, photons from $\lambda\bar{\lambda} \rightarrow J/\psi\gamma$ should still be detectable using one of the high-resolution detectors that have been proposed.⁵ High-energy resolution enables taking advantage of the near monochromaticity of the line which is

man rules a_λ effectively gets multiplied by a factor of 2 whereas v_λ vanishes. We will write the various amplitudes as if λ is a Dirac fermion, but will take the last remark into account when presenting cross sections for photinos. For supersymmetric particles such as photinos, Higgsinos, etc., the effective couplings in (1) have been expressed in terms of gauge couplings and scalar superpartner masses by Ellis *et al.*²

In models where the scalars associated with the left- and right-handed fermions, respectively, are nearly degenerate in mass the coupling v_f is small whereas it may be important for large mass differences [generally $v_f/a_f \approx (\bar{m}_L^2 - \bar{m}_R^2)/(\bar{m}_L^2 + \bar{m}_R^2)$].

With the effective Lagrangian (1) governing the coupling specifically between $\lambda\bar{\lambda}$ and a heavy-quark pair, we must prescribe the bound-state model before we can calculate the process $\lambda\bar{\lambda} \rightarrow ({}^{2S+1}L_J) + \gamma$ depicted in Fig. 1. We adopt the point of view of Rudaz⁴ that an adequate description of the process is provided by a nonrelativistic

Neutralino

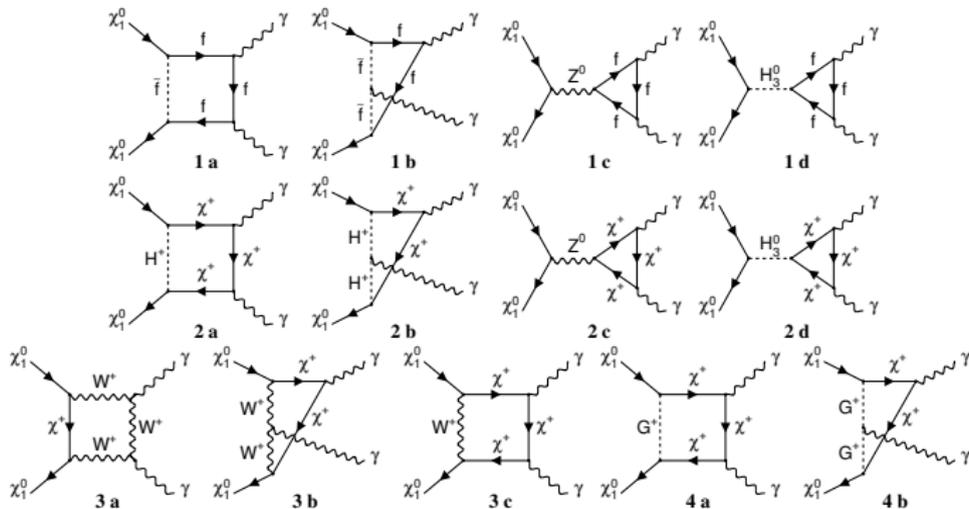
$$\chi\chi \rightarrow \gamma\gamma$$

$$E_\gamma = m_{DM}$$

$$\chi\chi \rightarrow \gamma Z$$

$$E_\gamma = m_{DM} \left(1 - \frac{M_Z^2}{4m_{DM}^2} \right)$$

Bergstrom, Ullio 97



Example: higgsino (H) and wino-like (W) LSP

	μ	M_2	$m_{\chi_0^1}$	$m_{\chi_0^2}$	$m_{\chi_{\pm}^1}$	$\sigma v_{\gamma\gamma}(\sigma v_{\gamma Z})$	$\sigma v_{WW}(\sigma v_{ZZ})$
H	139	1000	135.89	144.44	139.20	$1.0(3.4) \cdot 10^{-28}$	$2.1(1.4) \cdot 10^{-25}$
W	400	143	139.79	408.08	139.94	$2.0(10.9) \cdot 10^{-27}$	$3.4(0.0) \cdot 10^{-24}$

$$\text{BR}_{\gamma} = \frac{\sigma v_{\gamma\gamma} + 0.5\sigma v_{\gamma Z}}{\sigma v} \sim 0.08\%(0.2\%)$$

BARYOGENESIS WITHOUT GRAND UNIFICATION

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and Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg, Fed Rep Germany*

Received 8 March 1986

A mechanism is pointed out to generate cosmological baryon number excess without resorting to grand unified theories. The lepton number excess originating from Majorana mass terms may transform into the baryon number excess through the unsuppressed baryon number violation of electroweak processes at high temperatures.

The current view ascribes the origin of cosmological baryon excess to the microscopic baryon number violation process in the early stage of the Universe [1,2]. The grand unified theory (GUT) of particle in-

conserving baryon number violation processes as in the standard SU(5) GUT. (Baryon numbers would remain, if the baryon production takes place at low temperatures $T \lesssim O(100 \text{ GeV})$, e.g., after reheating

Gravitino

- Consistent cosmology with leptogenesis ($T_R \sim 10^9$ GeV), gravitino dark matter ($\Omega_{3/2} h^2 = 0.11$) and BBN ($\tau_{NLSP} \lesssim t_{BBN}$)

$$\Omega_{3/2} h^2 \simeq 0.27 \left(\frac{T_R}{10^9 \text{ GeV}} \right) \left(\frac{10 \text{ GeV}}{m_{3/2}} \right) \left(\frac{m_{\tilde{g}}}{1 \text{ TeV}} \right)^2$$

Buchmüller, Bolz, Brandenburg hep-ph/0012052

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- Gravitino meta-stable due to bilinear R-parity violation

$$\tau_{3/2}(\psi \rightarrow \gamma\nu) \simeq 10^{27} \text{ s} \left(\frac{\zeta}{10^{-7}} \right)^{-2} \left(\frac{M_1}{100 \text{ GeV}} \right)^2 \left(\frac{m_{3/2}}{10 \text{ GeV}} \right)^{-3}$$

Buchmüller, Covi, Hamaguchi, Ibarra, Yanagida 07; Bobrovski, Buchmüller, Hajer 1007.5007

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- Lifetimes of this order can be probed by indirect detection

Buchmüller, Ibarra, Shindou, Takayama, Tran 09; Ibarra, Tran, Grefe; ...

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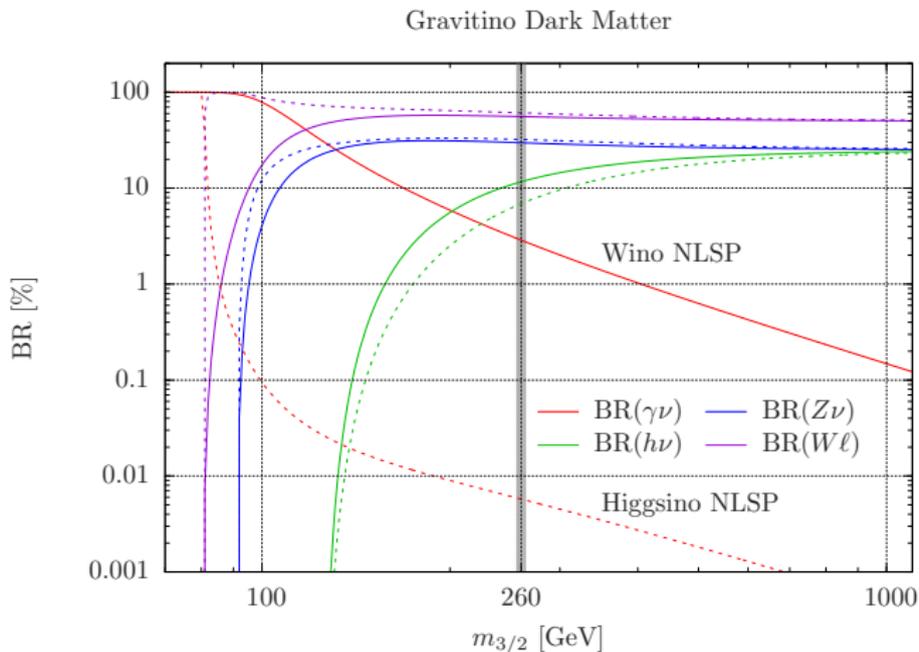
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- Lifetimes of this order can be probed by indirect detection

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- Two-body decay channels $\psi \rightarrow \gamma\nu, Z\nu, W\ell, h\nu$

$$\mathcal{L} = \underbrace{\frac{i}{\sqrt{2}M} (\bar{\chi}\gamma^\nu \gamma^\mu (D_\mu \phi)\psi_\nu + \text{c.c.})}_{\psi \rightarrow Z\nu, W\ell, h\nu} - \underbrace{\frac{1}{4M} \bar{\lambda}\gamma^\nu \sigma^{\mu\rho} \psi_\nu F_{\mu\rho}}_{\psi \rightarrow \gamma\nu}$$



Branching ratios of two-body gravitino decays for two representative examples

Wino NLSP: $M_2 = 1.1 m_{3/2}$, $M_1 = \mu = 10 m_{3/2}$

Higgsino NLSP: $\mu = 1.1 m_{3/2}$, $M_1 = 10 m_{3/2}$, $M_2 = 1.9 M_1$.

Branching ratio for $\psi \rightarrow \gamma\nu$ for Wino NLSP

$$\text{BR}_{\gamma}^{\text{max}} \simeq \frac{3\pi\alpha}{2\sqrt{2}G_F m_{3/2}^2} \simeq 3\% \text{ for } m_{3/2} \simeq 260 \text{ GeV}$$

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- Line + continuum

$$\frac{dJ}{dE} = \alpha \left(\delta(E - E_\gamma) + \frac{dN_{EG}}{dE} + \frac{1 - \text{BR}_\gamma}{N_\gamma \text{BR}_\gamma} \frac{dN_{cont}^\gamma}{dE} \right) + \beta \left(\frac{E}{E_\gamma} \right)^{-\gamma}$$

Continuum Gamma Rays

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- Extragalactic contribution

e.g. Bertone, Buchmüller, Ibarra

$$\frac{dN_{EG}}{dE} = \frac{\Omega_{DM} \rho_{c0}}{\sqrt{\Omega_M} (H_0/c) \bar{J}_\psi} \frac{E^{1/2}}{E_\gamma^{3/2}} \left(1 + \frac{\Omega_\Lambda}{\Omega_M} \left(\frac{E}{E_\gamma} \right)^3 \right)^{-1/2} \Theta(E_\gamma - E)$$

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- Continuum gamma spectrum (from PYTHIA)

$$\frac{dN_{cont}^\gamma}{dE} \equiv \frac{1}{\sum_{f \neq \gamma} \text{BR}_f} \sum_{f \neq \gamma} \text{BR}_f \frac{dN_f^\gamma}{dE}$$

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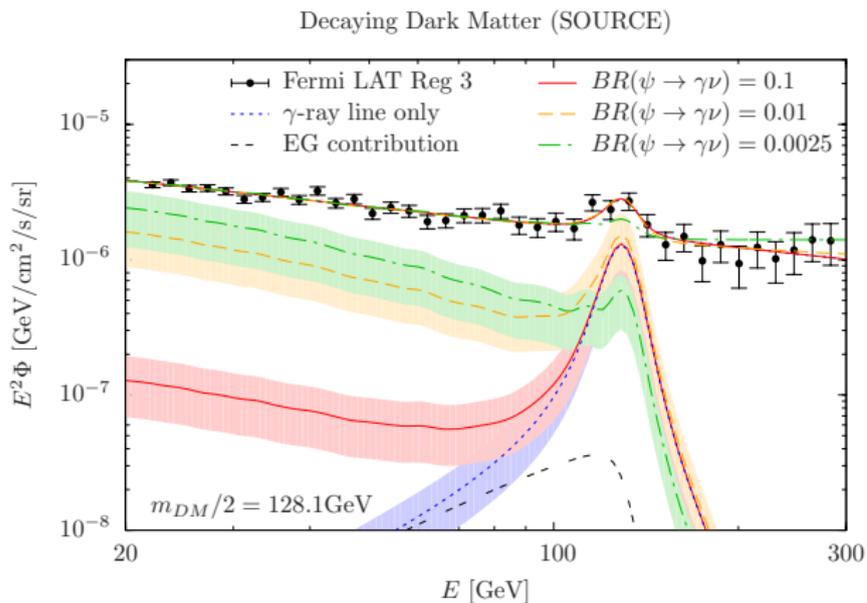
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- Independent of DM distribution and CR propagation

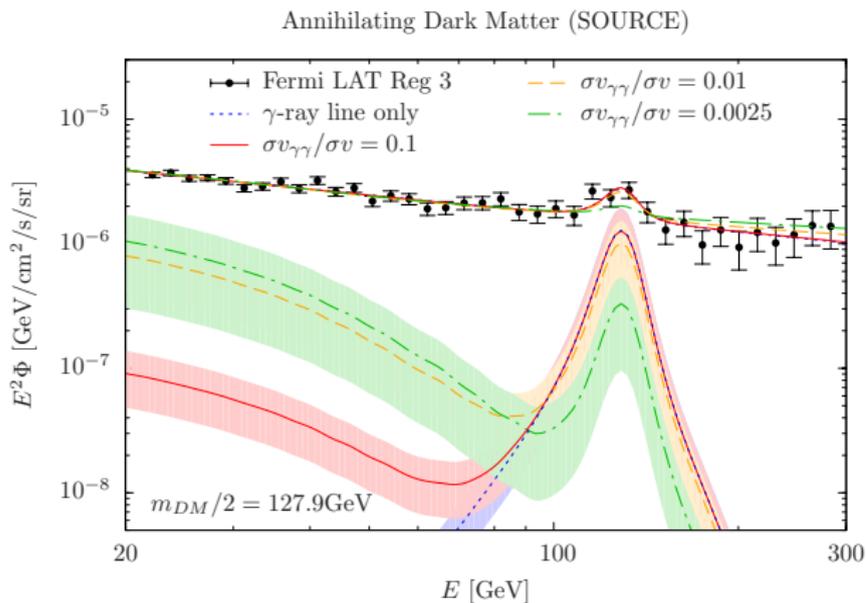
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$$\psi \rightarrow \gamma\nu, Z\nu, Wl, h\nu$$

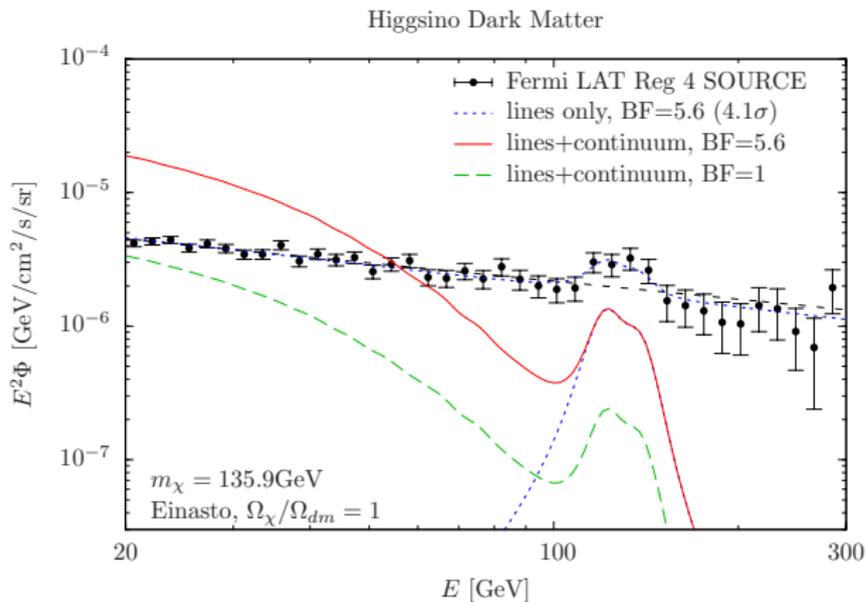


Continuum Gamma Rays

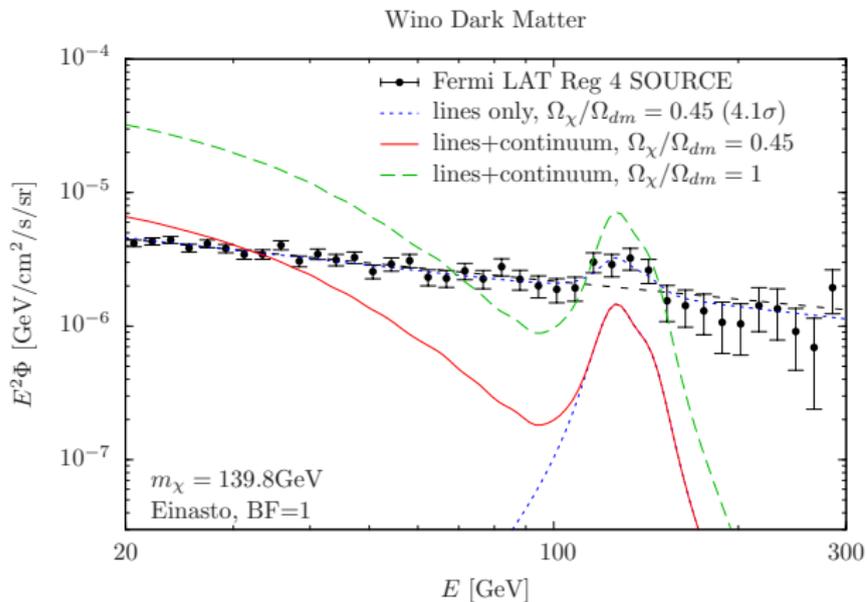
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Continuum Gamma Rays



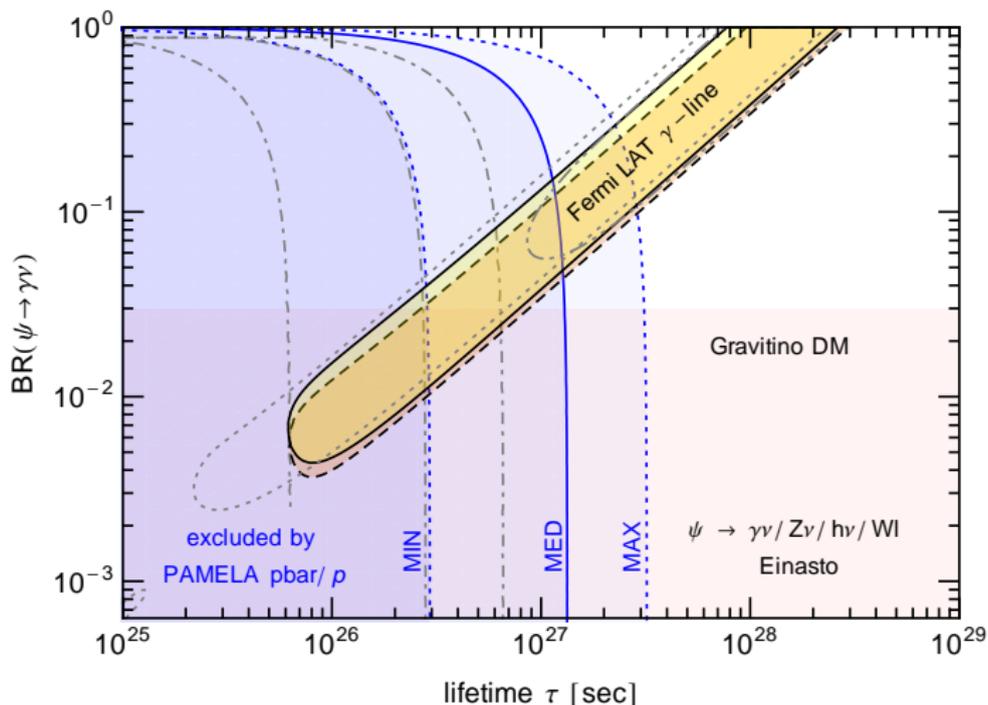
Continuum Gamma Rays



Continuum Gamma Rays + Antiprotons

$$\psi \rightarrow \gamma\nu, Z\nu, Wl, h\nu$$

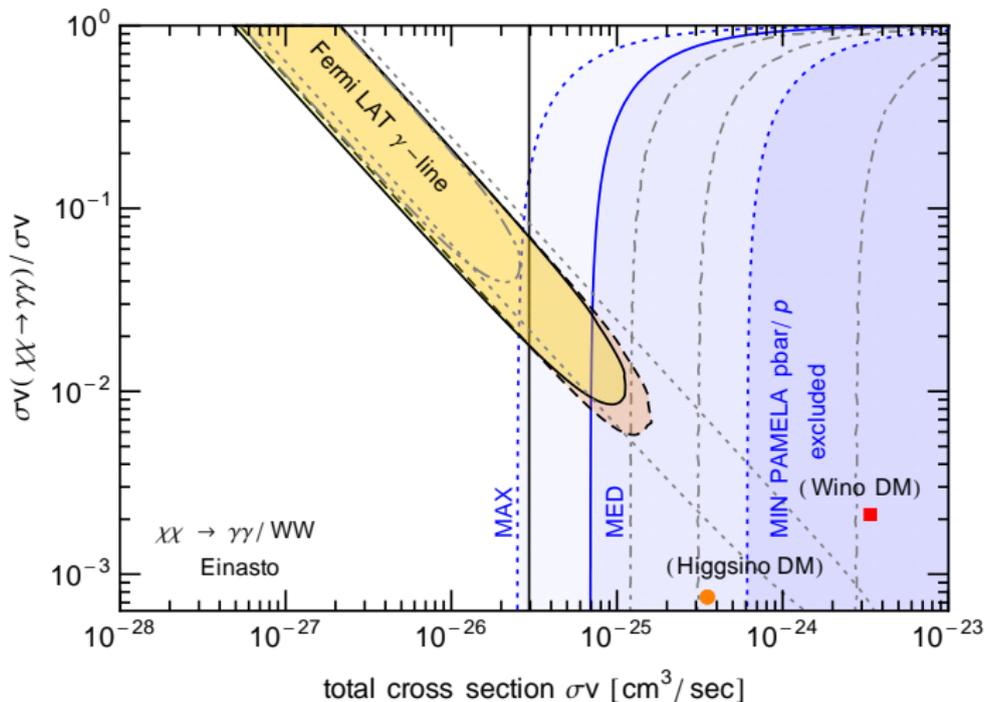
Decaying DM – SOURCE



Continuum Gamma Rays + Antiprotons

$$\chi\chi \rightarrow \gamma\gamma, WW$$

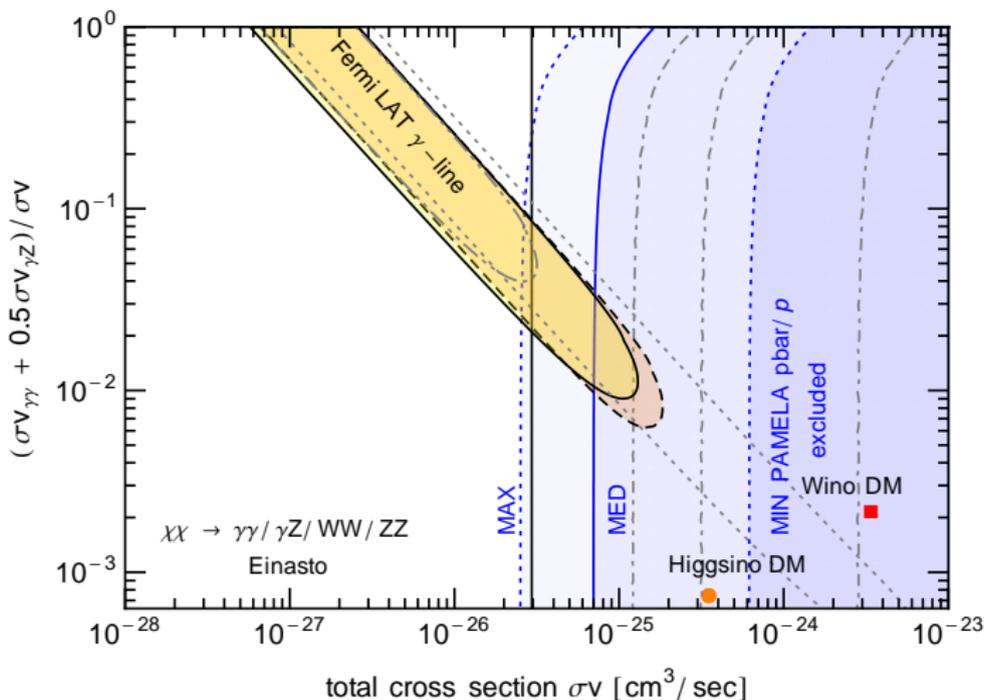
Annihilating DM – SOURCE



Continuum Gamma Rays + Antiprotons

$$\chi\chi \rightarrow \gamma\gamma, \gamma Z, WW, ZZ$$

Annihilating DM – SOURCE



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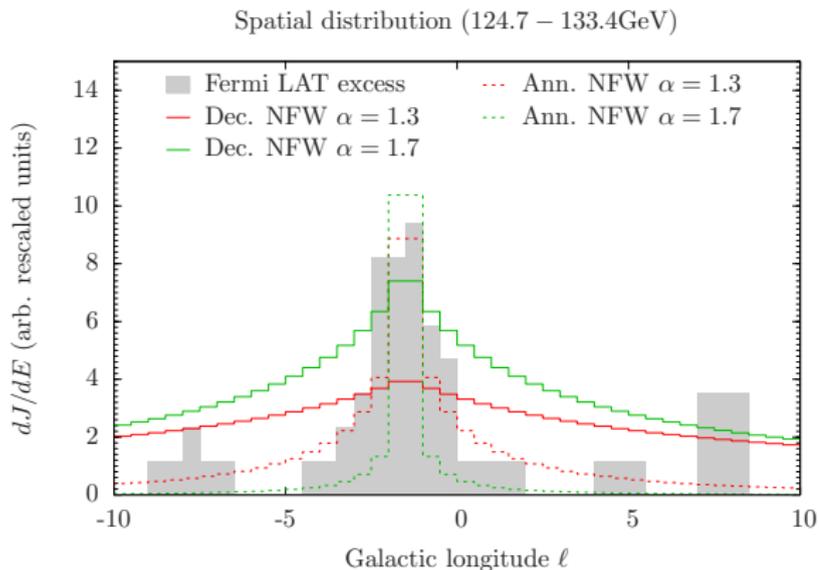
- Shape depends on line-of-sight integral over DM distribution (squared)

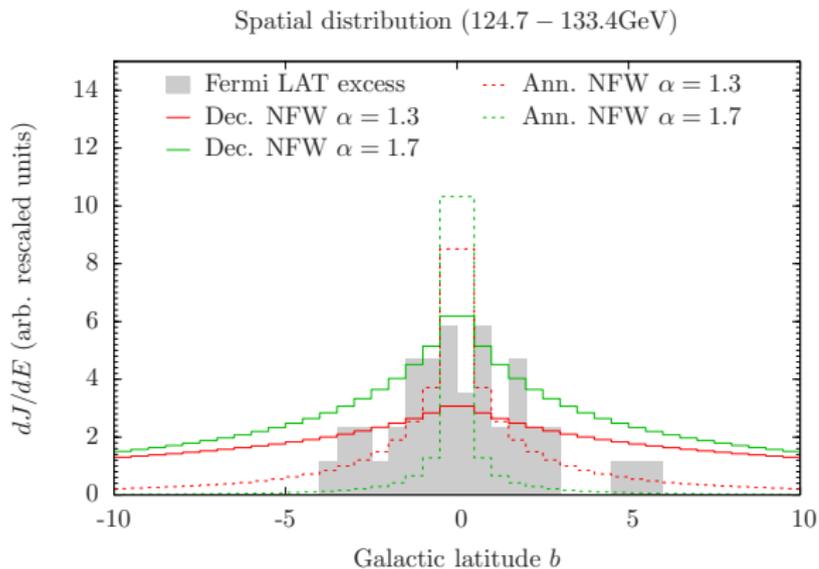
$$\frac{dJ_\gamma}{dE d\Omega} = \frac{1}{4\pi} \delta(E - E_\gamma) \left\{ \begin{array}{ll} \frac{1}{\tau_{\gamma\nu} m_{DM}} \int_{l.o.s.} ds \rho_{dm}(r) & \text{decay} \\ \frac{2\sigma_{\nu\gamma\gamma}}{m_{DM}^2} \int_{l.o.s.} ds \frac{1}{2} \rho_{dm}(r)^2 & \text{annihilation} \end{array} \right.$$

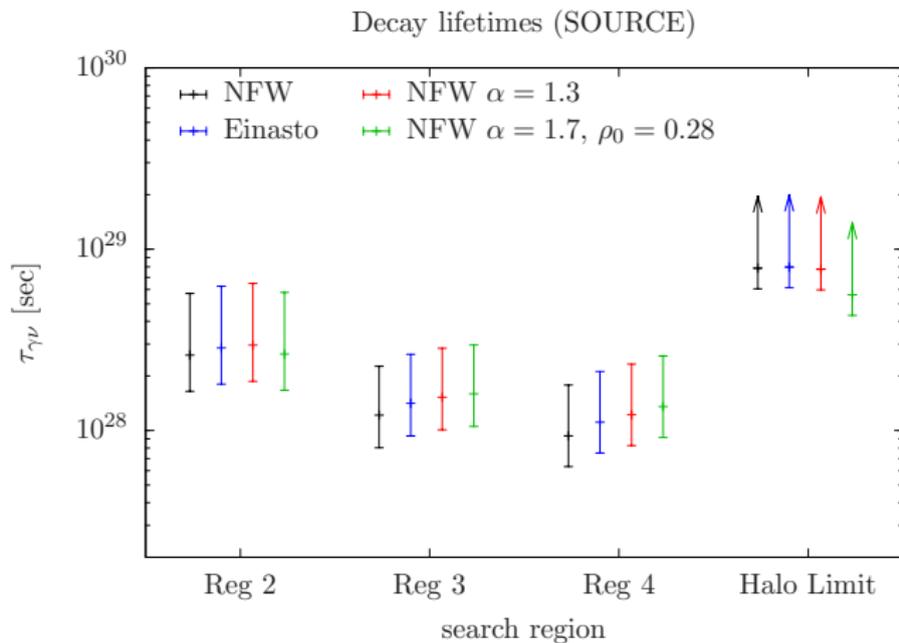
- NFW or Einasto profile

$$\rho_{dm}(r) \propto \frac{1}{(r/r_s)^\alpha (1 + r/r_s)^{3-\alpha}}, \quad \exp\left(-\frac{2}{\alpha_E} (r/r_s)^{\alpha_E}\right)$$

with $\alpha_E = 0.17$, $\alpha \geq 1$ and scale radius $r_s = 20\text{kpc}$







If the Fermi excess is real and due to DM...

- Continuum gamma rays severely constrain annihilating DM

$$\text{BR}_\gamma \gtrsim 0.5\%$$

- Independent of DM distribution and CR propagation
- Neutralino (higgsino/wino) ruled out as explanation of excess
- Gravitino with wino NLSP compatible
- Decaying DM would require enhanced DM density in the Galactic center region to fit the morphology of the excess

$$\Delta W = \mu_i H \ell_i, \Delta \mathcal{L} = B_i H_u \tilde{\ell}_i + m_{id}^2 \tilde{\ell}_i^\dagger H_d$$

$$\zeta_i = \frac{\epsilon'_i v_d + \epsilon''_i v_u}{v}$$
$$\epsilon'_i = -\frac{B'_i B + m_{id}^{2'} (\tilde{m}_{li}^2 - m_u^2)}{(\tilde{m}_{li}^2 - m_u^2)(\tilde{m}_{li}^2 - m_d^2) - B^2}$$
$$\epsilon''_i = \frac{B'_i (\tilde{m}_{li}^2 - m_d^2) + B m_{id}^{2'}}{(\tilde{m}_{li}^2 - m_u^2)(\tilde{m}_{li}^2 - m_d^2) - B^2}$$

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